MICRO-517 Lecture 7-9 Homework

1. Design of Cooke Triplet

The Cooke Triplet was designed and patented (GB 22,607) by British optical designer and inventor Harold Dennis Taylor (1862–1943) as a portrait lens in 1893 while working at T. Cooke & Sons of York. It can correct all seven Seidel (primary) aberrations while obtaining the desired focal length with only three optical elements, which is the minimum number of lenses required for such corrections. Taylor did not use any ray tracing technique in his design but totally relied on algebraic calculations.

The Cooke Triplet is widely considered as one of the most important lens designs in the field of photography with variants still in wide use today (Figure 1). Triplet lenses were the default lenses in old cameras of moderately wide to moderately tele focal length ranges. In modern times, compact cameras usually have a Triplet or a Tessar lens (a variant of the triplet lens). Owing to their lack of cemented surface, Cooke Triplet lenses are well known for their harsh-weather endurance and have been used on almost all expeditions to Antarctica and Mt. Everest.

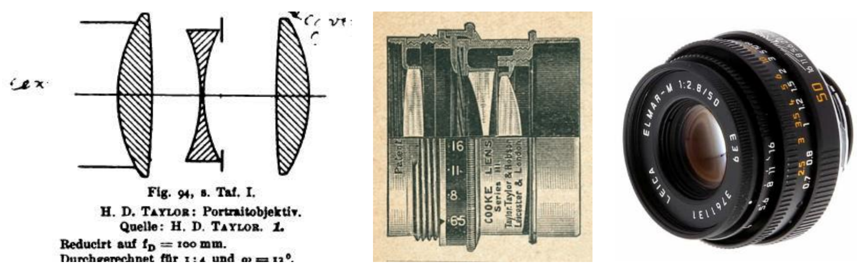

Figure 1. Left: An early illustration of the Cooke Triplet. Middle: Cooke Triplet, 1916. Right: Leica Elmar-M F/2.8, a triplet variant with 4 elements in 3 groups, 1994-2007.

In 1906, Taylor published his major written work, *A System of Applied Optics*, which is a 300-page development of the algebraic formulae for optical design. At the time, the great success of German designers with ray tracing triggered widespread use of such methods. With no computers, it was a highly laborious job for the designers of the time tracing rays on paper through various angles and positions over many surfaces. Taylor was quite negative on the concept of ray tracing. He argued that the time it takes for ray tracing is too long, but the results are not that different from testing the manufactured lens. He considered it a waste of time for lens designers, who should do more philosophical thinking than brute force calculations.

Today, with computers it takes a fraction of a microsecond to calculate the path of a ray through a spherical surface, which in the past would take an accomplished optical designer several minutes. It was once hoped that the design of lenses could be automated completely with the aid of computers. Soon it became clear that this was an illusion; the design of lenses makes the decisions from the designer necessary in many of its stages. Nevertheless, the efficient use of optical design software on the computer saves the designer a lot of time and effort. We therefore try to strike a balance between the roles of the designer and the computer in this homework.

1.1 Theory

The three lens elements in the Cooke Triplet provide eight degrees of freedom, which includes the three optical powers of the components, three shape factors (lens bending), and two air separations. Taylor's original reasoning of the triplet design is as follows:

1. The sum of the powers of the elements must be zero in order to have a zero Petzval sum (field curvature).
2. To have a low distortion and to correct the magnification chromatic aberration, the system must be nearly symmetrical. The possible solutions are then a negative lens between two positive lenses or a positive lens between two negative lenses. He realized later that the first solution leads to a better aberration correction.
3. To correct the longitudinal (axial) chromatic aberration (LCA), the central negative lens should be made with a flint glass and the two positive lenses should be made with crown glass.

While Taylor worked through algebra to correct all Seidel aberrations in his original design, here we emphasis the importance of a synergetic use of the designer's knowledge and the assistance from the optical design software. We follow part of Taylor's teaching in the layout and predesign phase to correct chromatic aberrations and field curvature and obtain a reasonable initial model, leaving the rest of the aberrations to ZEMAX OpticStudio optimization.

Let's begin with the paraxial layout of the triplet lens as shown in Figure 1Figure 2, assuming that the optical powers of the three lenses are K_1, K_2, and K_3, respectively, and the combined optical power is K, which is usually stated in the design specification. For reasons of symmetry, we set the stop at or very close to lens 2, which results in its chief ray height $\bar{h}_2 = 0$.

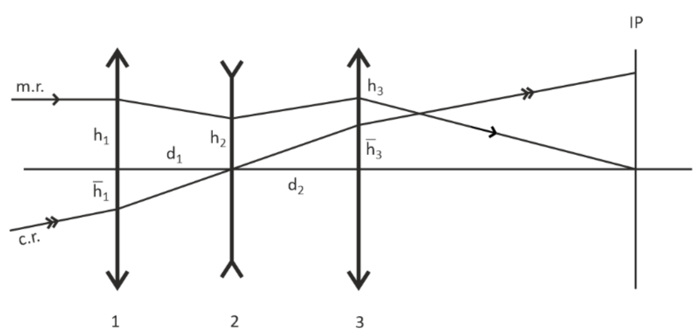

Figure 2. Layout of Cooke Triplet.

We list the equations that can be obtained from the paraxial layout:

$$K = K_1 + \frac{h_2}{h_1} K_2 + \frac{h_3}{h_1} K_3 \qquad \text{Power} \qquad (1)$$

$$P = \frac{K_1}{n_1} + \frac{K_2}{n_2} + \frac{K_3}{n_3} \qquad \text{Petzval} \qquad (2)$$

$$C_1 = \frac{h_1^2 K_1}{V_1} + \frac{h_2^2 K_2}{V_2} + \frac{h_3^2 K_3}{V_3} \qquad \text{LCA} \qquad (3)$$

$$C_2 = \frac{h_1 \bar{h}_1 K_1}{V_1} + \frac{h_2 \bar{h}_2 K_2}{V_2} + \frac{h_3 \bar{h}_3 K_3}{V_3} = \frac{h_1 \bar{h}_1 K_1}{V_1} + \frac{h_3 \bar{h}_3 K_3}{V_3} \qquad \text{TCA} \qquad (4)$$

By symmetry, we require the relation $\bar{h}_1 K_1 = -\bar{h}_3 K_3$. From trigonometry with the condition $\bar{h}_2 = 0$ we further have $\frac{\bar{h}_1}{d_1} = -\frac{\bar{h}_3}{d_3}$. Therefore, we have the relation $d_1 K_1 = d_2 K_3$.

We first attack Equation (4) further. We set our target to correct the longitudinal color aberration (LCA) such that $C_2 = 0$. The remaining targets is then K, P, and $C_1 = 0$. For simplification of discussion, we now define $\eta_2 = h_2/h_1$ and $\eta_3 = h_3/h_1 = V_3/V_1$. The lay-out equations now become:

$$K = K_1 + \eta_2 K_2 + \eta_3 K_3 \qquad (5)$$

$$P = \frac{K_1}{n_1} + \frac{K_2}{n_2} + \frac{K_3}{n_3} \qquad (6)$$

$$0 = \frac{K_1}{V_1} + \frac{\eta_2^2 K_2}{V_2} + \frac{\eta_3^2 K_3}{V_3} \qquad (7)$$

$$d_1 K_1 = d_2 K_3 \qquad (8)$$

Next, we continue with Equation (8). Through triangulation, we have $h_2 = h_1 - d_1 h_1 K_1$, so $\eta_2 = 1 - d_1 K_1$ and thus $d_1 K_1 = 1 - \eta_2$. With a similar triangulation and the lens equation, we also have $h_3 = h_2 - d_2\left(h_1 K_1 + h_2 K_2\right)$, so $\eta_3 = \eta_2 - d_2\left(K_1 + \eta_2 K_2\right)$ and $d_2 = \left(\eta_2 - \eta_3\right) / \left(K_1 + \eta_2 K_2\right)$. Plugging $d_1 K_1$ and d_2 into Equation (8), we have

$$\left(1-\eta_2\right) K_1 + \eta_2\left(1-\eta_2\right) K_2 + \left(\eta_3 - \eta_2\right) K_3 = 0 \qquad (9)$$

Subtracting Equation (5) from Equation (9) leads to

$$K = \eta_2 K_1 + \eta_2^2 K_2 + \eta_2 K_3 \qquad (10)$$

Furthermore, from Equations (5) and (7) we obtain

$$K = \left(\eta_2 - \frac{\eta_2^2 V_1}{V_2}\right) K_2 + \left(\eta_3 - \frac{\eta_3^2 V_1}{V_3}\right) K_3 \qquad (11)$$

which, by plugging in $\eta_3 = V_3/V_1$, becomes

$$K_2 = K \Big/ \left(\eta_2 - \eta_2^2 V_1/V_2\right) \qquad (12)$$

Inserting Equation (12) into Equations (5) and (10), and rearranging Equation (7) by multiplying both sides with V_1, we have the following three equations

$$\begin{gathered} K_1 + \beta_3 K_3 = K - \eta_2 K_2 \\ K_1 + K_3 = K/\eta_2 - \eta_2 K_2 \\ K_1 + \eta_2^2 K_2/\beta_2 + \eta_3^2 K_3/\beta_3 = 0 \end{gathered} \tag{13}$$

for solving three unknowns K_1, K_2, and K_3 with the required K, and three free choices of the height ratio η_2 and the Abbe ratios of $\beta_2 = V_2/V_1$ and $\beta_3 = V_3/V_1$. The height ratio η_2 usually ranges from 0.7 to 0.9, and $\eta_2 = 0.8$ works fine according to the advice of German physicist and mathematician Max Berek (1886–1949).

As a last step we determine the distances d_1 and d_2. From $\eta_2 = 1 - d_1 K_1$ and $d_1 K_1 = d_2 K_3$ it follows that

$$\begin{gathered} d_1 = (1-\eta_2)/K_1 \\ d_2 = (1-\eta_2)/K_3 \end{gathered} \tag{14}$$

At this point, we must make our choice of glass in order to proceed. Once the type of glass is fixed, β_2 and β_3 are set, and the individual optical powers are determined from the required K, as well as the Petzval sum P. It is advisable from the perspective of field curvature to choose SK glass (high n, high V) for the outer lenses and LF glass (moderate n, low V) for the inner lens. It is, therefore, to be understood that the performance of a Cooke Triplet is dependent on the choice of the glass. However, this can be done by the computer, so the initial choice is not that critical.

Alternatively, the solution can be found by fixing K_2, and K_3 first and then work out K_1, β_2, and β_3. This approach gives more insight into the choice of glass. It is important to note that higher power lenses have more aberration and require higher assembly precision. So we should use the lowest possible optical power in each lens element.

1.2 Design Task

Now that we have acquired a reasonable grip of ZEMAX OpticStudio, let's design a Cooke Triplet and optimize it based on the calculations using the theory introduced above. The design of Cooke Triplet is often considered the high point in an optical design course.

The lens should have an effective focal length of 120 mm and an aperture of F/5. Standard F.d.C. colors are to be used in the design and color correction. Two designs should be made with the half field angle of $w = 20°$ and $w = 5°$, and their performance should be compared in terms of spot size and Seidel coefficients.

1.3 ZEMAX Optimization

It is advisable to avoid Global Searching if a rationally obtained initial structure is established in ZEMAX OpticStudio. The Global Searching is useful when we have totally no idea what initial structure to feed to the optimizer. It is time-consuming, and the results are often unsatisfactory, particularly when the degree of freedom is high.

An overall principle for efficient optimization is to begin with the most influential few parameters so that the optimizer will not get lost in a space of too many degrees of freedom. Then a few less influential parameters can be open to optimization. Finally the least influential parameters can be open for tine tuning. In a lens system, the surface curvature radii are the most influential ones. Air distances between the lens elements are much less influential, while the lens thicknesses are the least influential ones. Another principle when working with ZEMAX optimizer over a predesigned structure is to perform local optimization first and then use the “Hammer” optimizer to find better solutions nearby with glass substitute.

Based on the above principles, the following procedure is recommended for obtaining a reasonable optimization result within a bounded time:

1. Set up the design in ZEMAX lens datasheet using three flat plates of your chosen glass according to the specification. Set the distances between the component lenses (d_1 and d_2) according to the predesign. Use a thickness of 2 mm for the central negative lens and an approximate thickness (say 5 mm) for the positive lenses to ensure the edge thicknesses are positive. Use an approximate distance for the last distance before imaging (the effective focal length can be used here). Make surface 4 the STOP.

2. Use “Element Power” solve on surfaces 2, 4, and 6 with the power obtained from the predesign. Now we have three asymmetric lenses with their first surface flat and second surface spherical. To restore the symmetry, copy-paste the radius value from surface 2 to surface 1 and change its sign. Now surface 2 should become (almost) flat. Next, enter the radius value of surface 4 times 2 as the radius of surface 3. Now the system should look nearly symmetric. Check the effective focal length (EFFL in the status bar at the bottom), which should be not too far from the required value.

3. Click “Quick Focus” and bring the image plane to the best focus. Theoretically, the focus of the image plane can be automatically maintained by setting the “Marginal Ray Height” solve in the last distance with a value of 0. However, the use of this solve in optimization often leads to an unpredictable result. So let’s avoid the use of it here. However, there will be a conflict between a fixed image plane and a required effect focal length during optimization. Therefore, multiple “Quick Focus” will be necessary to bring the image plane back to focus during the process of optimization.

4. Now we can play with visual optimizer and gain some insight on the roles of the radii and the thicknesses in the aberrations. First, set the radii of surfaces 1, 3, and 5 to be variable. With the layout, spot diagram, and Seidel diagram visible, click “Visual Optimizer” in the Optimize menu tab and move the three sliders to change the shape of the lenses and see how the Seidel coefficients and the spots change. Owing to the

"Element Power" solve, the effective focal length will remain nearly constant. You may need to change the range of the parameters to explore a larger parameter range. If you are happy with the results, click "Keep and Exit", otherwise just click "Exit". Next, click "Remove All Variables" and set the two air thickness (of surfaces 2 and 4) to be variable and explore their influence. You can explore the role of the three glass thicknesses similarly.

5. Use "Quick Focus" to bring the image plane back to focus in case there is any slight change I the effective focal length.
6. Prepare for the optimization by setting up the merit function using the "Optimization Wizard". Once the default merit function is created, add the following two more constraints on top of the default ones with weight of 1.0: "EFFL" with wave = 2 and target = 100, which means the effective focal length at wavelength 2 should be 100 mm, and "AXCL" with wave1 = 1, wave2 = 3, zone = 0, and target = 0, which means the axial color focal shift between wavelength 1 and 3 should be 0 using paraxial calculation (rather than real ray, specified by zone = 0).
7. We can now begin local optimization. Click "Remove All Variables", make all radii variable, including those with "Element Power" solve, and click "Optimize!" with infinite cycles. Wait until the merit function does not change in a few seconds or so and click "Stop". If you make the layout and the spot diagram windows visible before the optimization and enable "Automatic Update", you can observe the optimizer at work in real-time. Click "Quick Focus" to remove any slight defocus. Next, make all thicknesses except the last one variable (while keeping the radii variable) and optimize until the merit function does not change in a minute or so and clock "Stop". Click "Quick Focus" again to remove any slight defocus.
8. By now the design should be relatively optimized based on the glass chosen. To further finetune the performance, the "Hammer" optimizer can be used. Keep all the variables and set the solve of the three glass fields to "Substitute" to enable glass substitution during optimization. Click "Hammer Current" to further optimize the design (click "Start" rather than "Automatic" button to start). You can monitor the optimizer at work by making the relevant windows visible before the optimization begins and enable "Automatic Update". The "Hammer" optimizer will run indefinitely until the user clicks "Stop". This should be done when the merit function does not change over a few minutes, which can be checked from the "Status: Last Save:" display field.

After the above procedure, the design should be well optimized (see Figure 3 and Figure 4 for examples of optimized design layout and selected performance analysis). When optimizing a design, we should realize that there is no "the best" but only "good enough" solution. One could never know whether a particular local minimum found by the computer is the global minimum or not. We also need to note that the Global Search and the Hammer optimizer are of a stochastic nature and the outcome may not be repeatable.

So probabilistically speaking, good luck!

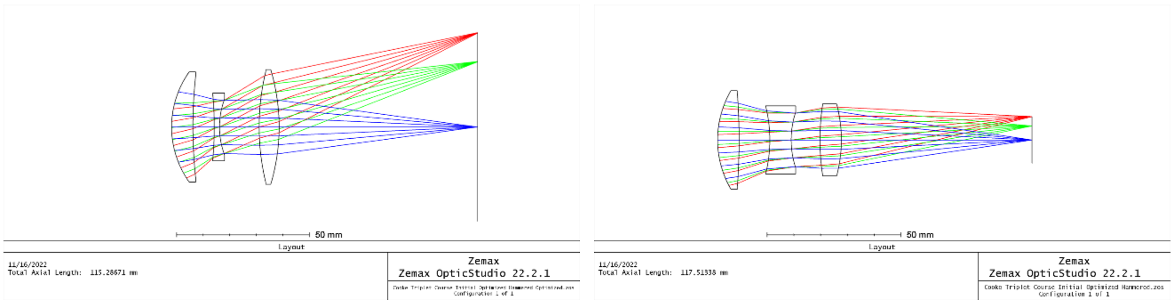

Figure 3. Cooke Triplet design examples. Left: 20° half field angle. Right: 5° half field angle. Upper row: design layout.

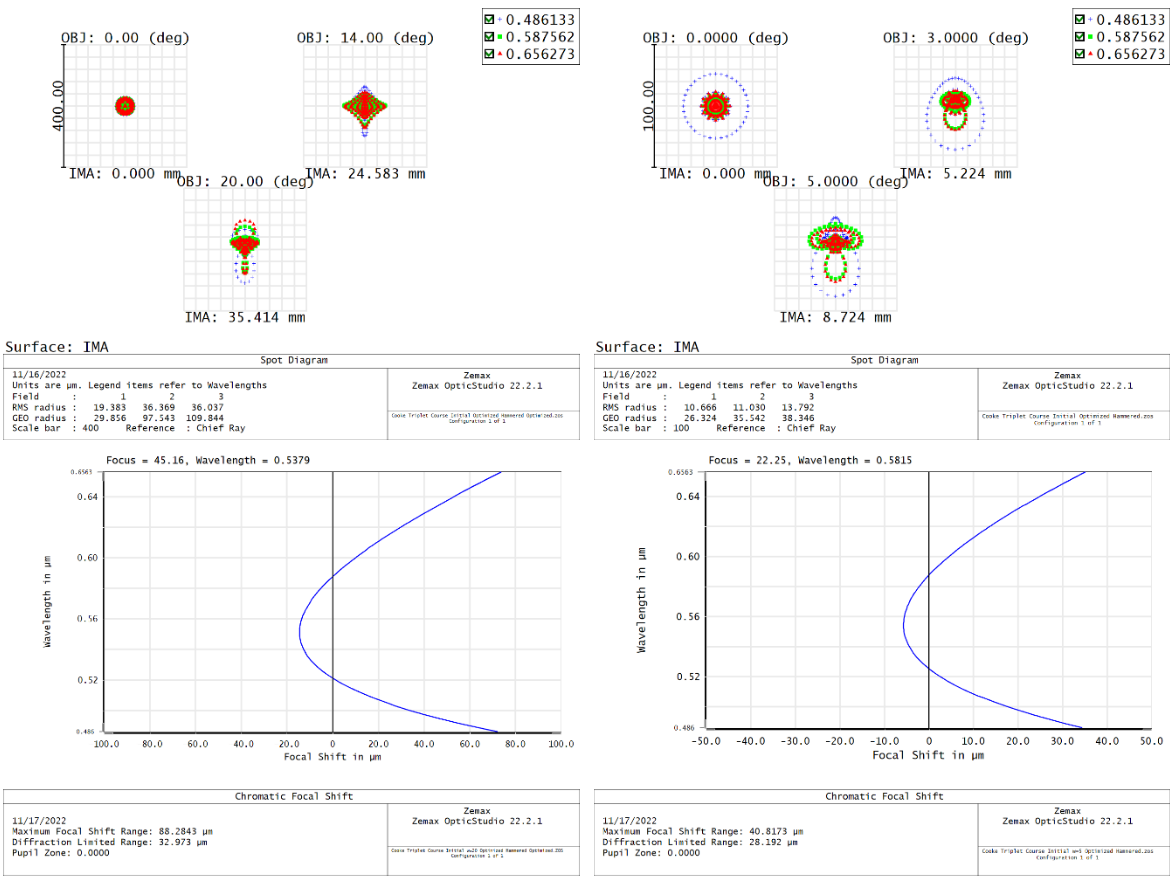

Figure 4. Selected performance analyses of the Cooke Triplet designs after optimization. Left column: 20° half field angle. Right column: 5° half field angle. Upper row: spot diagram. Lower row: chromatic focal shift.

2. Wave optical investigation of the design

With the optimization of your design completed, investigate the PSF and MTF of the design. Use FFT methods with sufficient resolution. The Huygens PSF and MTF methods are in general more accurate. However, the accuracy comes at a cost of significantly longer computational time. So unless the design has an aperture greater than F/1.5 or NA of 0.7, FFT methods should be used with good trust.

For the PSF calculation, use “FFT PSF”. Use “False Color” option in the “Show As” field in “Settings”to generate a more useable 2D map of the PSF. Check the “Use Normalize” field in Settings so that we see a meaningful plot with large aberrations. Use a sufficient sampling such as 1024 × 1024 for the computation to reveal the PSF in its entirety. Generate PSF for “All Wavelengths”. See Figure 5 for an example of PSF images.

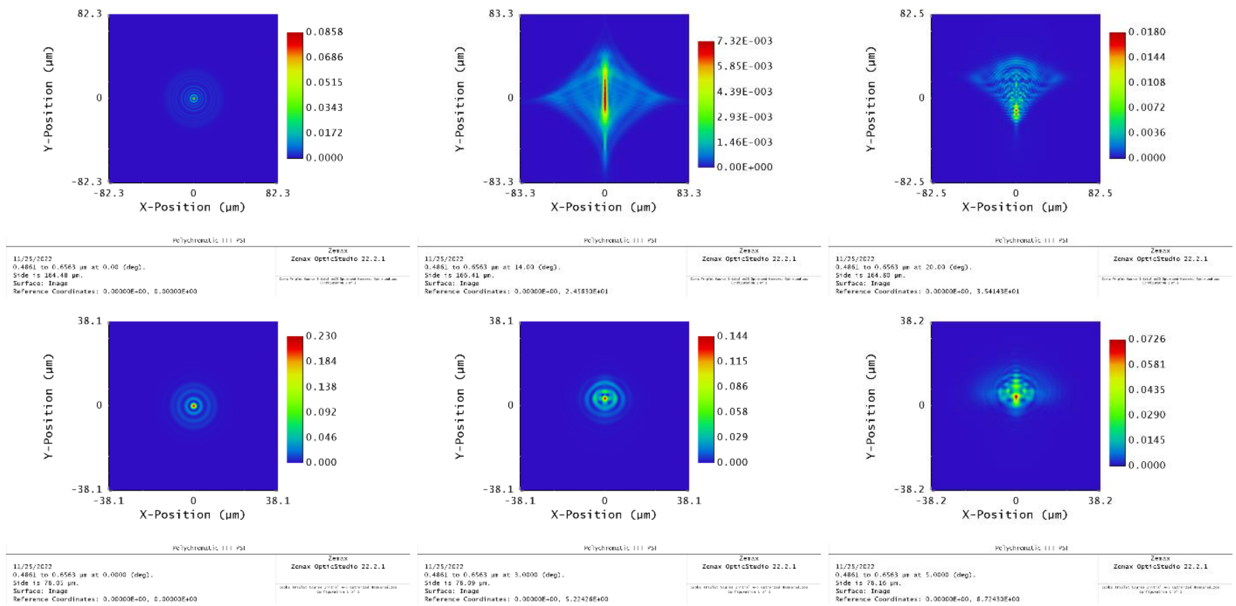

Figure 5. Example PSF analyses of the Cooke Triplet designs after optimization. Upper row: 20° half field angle. Lower row: 50° half field angle. Left column: Field 1. Middle column: Field 2. Right column: Field 3.

For the MTF calculation, use “FFT MTF”. Check the “Show Diffraction Limit” field in the Settings to show the theoretical limit of MTF. Use a sufficient sampling such as 256 × 256 to ensure the accuracy of the MTF. Generate the MTF for “All Wavelength” and “All Field”. See Figure 6 for an example of MTF plots.

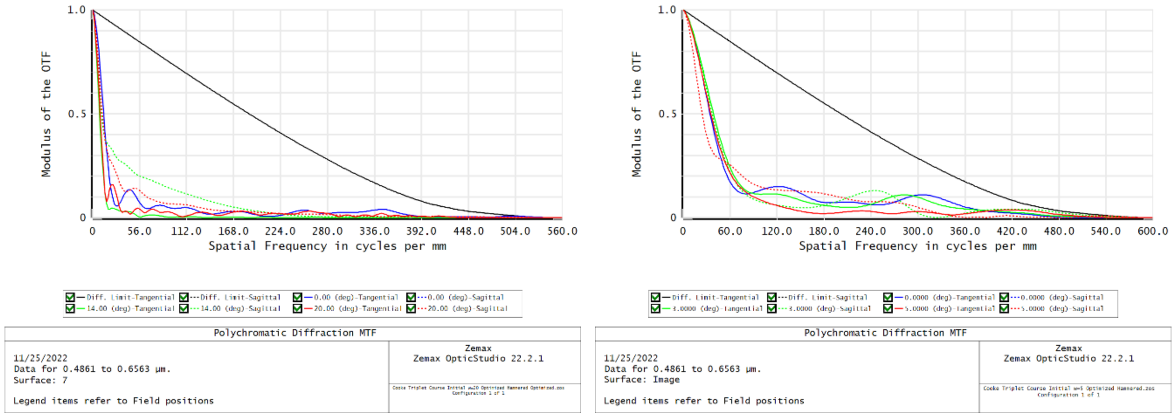

Figure 6. Example MTF analyses of the Cooke Triplet designs after optimization. Left: 20° half field angle. Right: 5° half field angle.

These results reveal the compromise in the design of triplet lenses: we can have either a large field of view with lower optical resolution or a high optical resolution at a smaller field of view. It takes more degree of freedom to correct the aberrations for a large field of view at high resolution.

4. Submission

Submit all relevant ZEMAX files with a brief report outlining your calculations to determine the initial lens structure, and showing the optimized lens layout, the spot diagram, Seidel diagram, PSF, and MTF of the two designs, as well as a comparison summary of the two designs.